Prediction of bandgaps in membrane-type metamaterial that attached to thin plate

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Prediction of bandgaps in membrane-type metamaterial that attached to thin plate

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ABSTRACT

The bandgap property of the membrane-type metamaterial is closely related to the tension stress that applied to the membrane. Such type of metamaterial is composed by periodically allocated prestressed membranes that decorated with lump masses. In this paper, we proposed an analytical method to analyse the bandgap location and width of membrane-type metamaterial when attached to thin plate structure. This method enables the bandgap prediction of such structure by adjusting the tension stress directly. Accuracy of the model is verified by numerical model and the results show that the results given by analytical model are basically consistent with the simulation. The effect of tension and attached mass magnitude on bandgap location and width is also examined. The bandgap width is in direct proportion to tension stress and mass magnitude.

Keywords: Membrane-type, Metamaterial, Bandgap, Prediction
I-INCE Classification of Subject Number: 47

1. INTRODUCTION

Metamaterials are manually engineered materials that have peculiar effective material properties that not available in natural materials [1]. For acoustic/elastic metamaterials, their ability in attenuating the transmission of wave attracted many research efforts. Membrane-type metamaterial is one type of acoustic metamaterial. It was first proposed by Yang et al in 2008 [2]. For acoustic metamaterials, local resonance is demanded in generating bandgaps. The unit cells of membrane-type metamaterials are normally formed by elastic membranes attached with lumped mass and stretched over rigid frames. Such structure is schematically equivalent to a mass-spring resonator and thus similar to other types of metamaterial, it possesses local resonance bandgaps in resonance frequency range. The location of bandgap is decided by the resonance frequency of the unit membrane-type resonator.

The membrane-type metamaterial can be used in sound isolation, energy harvesting and vibration absorption and the corresponding effectiveness of application were studied [3, 4, 5]. Mei et al [6] used specially designed platelets to decorate the membrane and illustrated that their design with single layer membrane can absorb 86% of the acoustic wave around the resonance peak frequency whilst the double layer design can absorb 99%. Naify et al [7] examined both single-cell resonator and multi-cell resonator's behaviour at low frequencies (below 200Hz) and proved that the membrane-type resonators can achieve much higher transmission loss than the prediction of mass-law. Otherwise, Liang Sun conducted experiments to study the membrane-type resonator's

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capability in structural vibration control. The results showed that the membrane resonator can effectively reduce rectangular plate vibration magnitudes by up to 42 dB [8].

In addition, for actual application, the operation frequency of the metamaterial may be various according to the incident wave. Therefore, to enable the agile bandgap location, some researchers have investigated the potential methods to tune the bandgap of membrane-type metamaterial. Langfeldt et al [9] proposed an inflatable membrane structure, through which the stress within the membrane can be adjusted by the extent that the membrane is inflated and thus the bandgap location of the metamaterial. Chen et al [10] proposed a membrane-type metamaterial that was magnetically controllable. The resonant frequencies of the unit cell can reach up to about 64% higher level if increase the input magnetic field.

In the aforementioned researches about membrane-type metamaterials, the focus points were mainly about the effectiveness in different application fields or tuning of the bandgap properties. The prediction of the resonance frequencies of the designed membrane resonators before actual fabrication were either neglected to be mentioned in the papers or conducted by finite element methods (FEM) through software. The prestress level on membrane is acknowledged as the main factor that affect the equivalent stiffness within unit cell yet the detailed relation between tension stress and resonance frequencies were not included. For application of membrane-type metamaterial, it is necessary to know the possible bandgap properties in advance and enable the proper design of membrane resonator parameters (tension stress, mass magnitude etc.) in accordance to the demand. Moreover, not many researches have studied the membrane-type metamaterial's bandgap properties when they are distributed periodically.

In this paper, we proposed a theoretical model that can provide prediction of bandgap properties of membrane-type metamaterial when it is applied to thin plate structure. Through the model, the changing of bandgap properties of the membrane-type metamaterial on thin plate through tuning the tension stress and attached mass magnitude can be revealed directly. Verification of accuracy is carried out by commercial FEM software COMSOL Multiphysics.

2. MODEL AND FORMULATIONS

2.1 Resonance frequency of membrane-type resonator

The resonance frequency of membrane-type resonator can be predicted by the Rayleigh-Ritz method. This method has been examined to be accurate enough [11]. The structure of a membrane-type resonator can be schematically depicted as in Figure 1. The length and height of the host frame and membrane are denoted as L, H, l and h respectively. The mass is assumed to be concentrated at a certain point and the coordination is (a, b). The frame thickness is neglected in this method as it is assumed to provide constrains to the membrane. In the actual design, the frame material should be relatively light and rigid to prevent causing extra influence.

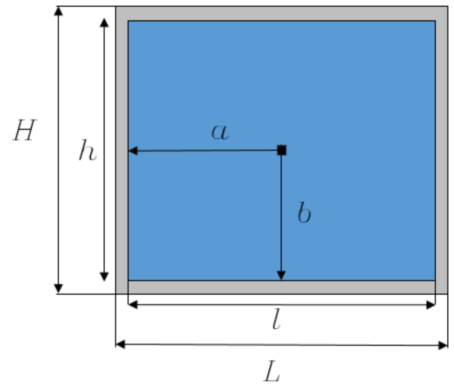

Figure 1. Configuration of membrane-type resonator

The strain energy and kinetic energy can be given as [11]:

$$S_{max} = \frac{1}{2}\iint D\{(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})^2 - 2(1-v)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - (\frac{\partial^2 w}{\partial x \partial y})^2]\}dxdy + \frac{1}{2}\iint T[(\frac{\partial^2 w}{\partial x^2})^2 + (\frac{\partial^2 w}{\partial y^2})^2]dxdy \quad (1)$$

$$K_{max} = \frac{\omega^2}{2}\{\iint m_s w^2(x,y)dxdy + m_R(a,b)\, w^2(a,b)\} \quad (2)$$

where D is the bending stiffness of the membrane:

$$D = \frac{Et^3}{12(1-v^2)} \quad (3)$$

E, t and v are the Young's modulus, thickness and Poisson's ratio of the membrane respectively. T is the tension stress per unit length on membrane, $m_R(a,b)$ is the mass located at coordinate (a,b). m_s is the membrane mass per unit area. $w(x,y)$ and $w(a,b)$ are the transverse displacement of the membrane and mass at the coordination indicated. ω^2 is the natural frequency of the resonator.

According to equation (1) and (2), it yields the natural frequency as:

$$\omega^2 = \frac{2U_{b,max}}{\iint m_s w^2(x,y)dxdy + m_R(a,b)\, w^2(a,b)} \quad (4)$$

A modal shape function is needed to be substituted into the equation and work out the natural frequency. In this method, the function is assumed as [11]:

$$w(x,y) = A_{mn}\sin\left(\frac{\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)\sin\left(\frac{\pi y}{h}\right)\sin\left(\frac{n\pi y}{h}\right) \quad (5)$$

For membrane-type resonator, we mainly focus on the first order resonance frequency. Substitute the corresponding shape function into equation (4), then the membrane-type resonator's lowest natural frequency is given as:

$$\omega_{11} = \frac{1}{2\pi}\sqrt{\frac{\frac{\pi^4 D}{4l^3 h}(3h^4 + 3l^4 + 2l^2h^2) + \frac{3(l^2+h^2)T\pi^2}{16lh}}{\frac{9lhm_s}{64} + m_R sin^4(\frac{\pi a}{l})sin^4(\frac{\pi b}{h})}} \quad (6)$$

2.2 Dispersion relation

To obtain the dispersion relation curve of the system, the Plane Wave Expansion (PWE) method is used. This method is proved to be able to provide accurate prediction for dispersion relation of thin plate structure periodically attached with spring-mass resonators [12]. As the membrane-type metamaterial can be simplified as spring-mass resonators, we used the PWE method to predict the bandgap property.

As illustrated in former session, the first order resonance frequency of the membrane resonator can be derived, and the mass magnitude is already known. As a result, according to the equation $\omega_n = \sqrt{\frac{k_R}{m_R}}$, the equivalent stiffness of the unit cell can be obtained.

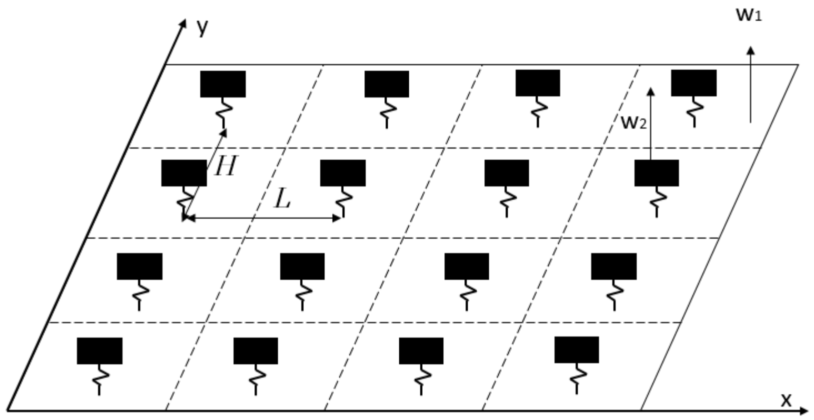

Figure 2. Configuration of thin plate periodically attached with membrane-type metamaterial

The system configuration can be simplified as Figure 2. According to the dimension of membrane resonator frame, the outer dimension of a single unit cell is $L \times H$.

The equation of motion for the system can be given by these equations:

$$\begin{cases} D\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w_1(x,y) - \omega^2 m_s w_1(x,y) = \sum_R f_1(X,Y)\delta[(x-X, y-Y)] & (7) \\ -\omega^2 m_R w_2(X,Y) = f_2(X,Y) & (8) \end{cases}$$

where (x, y) and (X, Y) are the coordinates of points on the plate and the location of resonators, $w_1(x,y)$ and $w_2(X,Y)$ are transverse displacement of plate and resonator at different points, f_1 and f_2 are forces that applied on thin plate and resonator masses, and δ is Dirac function. Applying Bloch theorem and change the coordination of points into lattice vector, then the equation of motion can be transformed into matrix form [12]. The equation can then be solved by applying a truncation in the plane wave number series. The mass magnitude and equivalent stiffness are integrated into the equation and can therefore reveal the effect to bandgap structure.

3. RESULTS AND DISCUSSION

3.1 Bandgap of the infinite structure

In PWE model, the structure is assumed to apply with periodical boundary conditions, as a result, the calculated bandgap is for the infinite structure. In the section, examples of membrane resonators with various mass and tension stress are considered. The corresponding change of bandgap location and width are revealed.

Define the parameter of the membrane resonator as indicated in Table 1. The mass magnitude is 2.7g and attached in the middle of the resonator. The materials for membrane and frame are chosen as silicon rubber and epoxy respectively. The parameters of the plate that the resonators are attached to are defined as: Young's modulus $E = 200GPa$, Poisson's ratio υ =0.3 and density ρ =7850 kg/m^3. Also, by this method, the thickness of the plate has to be smaller than the flexural wavelength that propagate in the plate. We define the thickness of the plate as 2mm in this case.

Table 1. Parameters of membrane resonator

Membrane		Frame		Mass	
Young's modulus (MPa)	1.9	Young's modulus (GPa)	2.65	Magnitude (g)	2.7
Poisson's ratio	0.48	Poisson's ratio	0.41	Radius (mm)	5
Density (kg/m^3)	980	Density (kg/m^3)	1100	Height (mm)	4
Thickness (mm)	0.2				
l (mm)	50				
h (mm)	50				

In order to examine the effect of tension stress, the stress that applied to the membrane is defined as 2MPa, 3MPa, 6MPa and 10MPa respectively. The bandgap structures of the resonator with different stress are given in Figure 3.

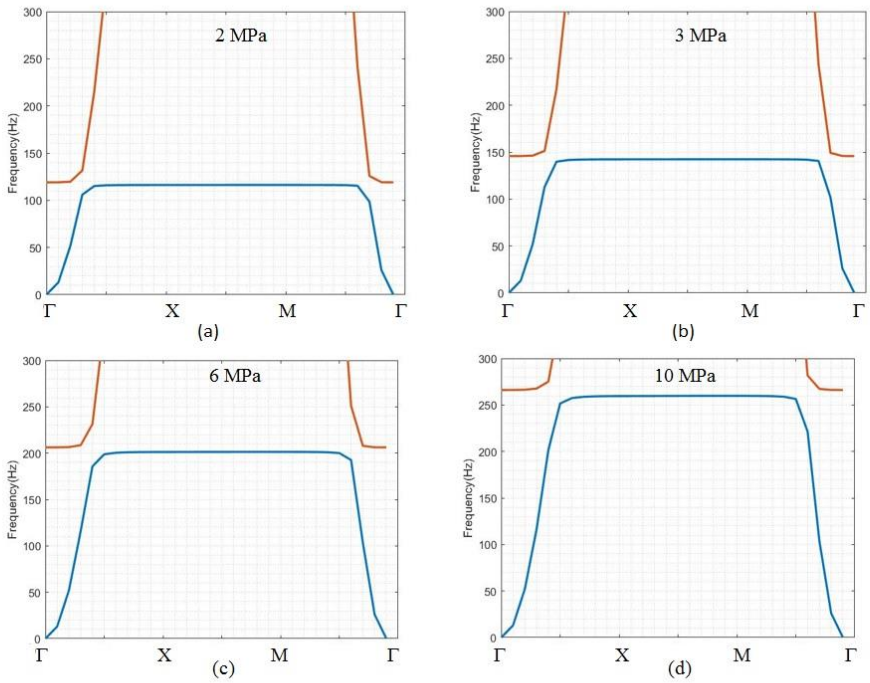

Figure 3. Bandgap structure of membrane-type metamaterial applied with (a) 2MPa, (b) 3MPa, (c) 6MPa and (d) 10MPa stress.

According to Figure 3, the bandgap shifts to higher frequency range along with the increment of stress. When 2 MPa stress applied to the membrane, a full narrow bandgap exist between 116.3 – 119.1 Hz. Once the stress increased to 10 MPa, the bandgap shifts to 259.8 – 266.1 Hz. The changing trend of bandgap fits the pattern of bandgap change for membrane-type metamaterial. Verification for model accuracy will be conducted in later section. Detailed location of bandgap and width are given in Table 2.

Table 2. Bandgap property of membrane-type metamaterial applied with various stress

Stress (MPa)	2	3	6	10
Upper edge (Hz)	119.1	145.8	206.1	266.1
Lower edge (Hz)	116.3	142.4	201.3	259.8
Band width (Hz)	2.8	3.4	4.8	6.3

Otherwise, the adjustment of mass magnitudes are also examined. We define the mass magnitude as 2.7g, 5.4g and 10.8g and the stress is set at 2 MPa. The corresponding bandgap structure is given in Figure 4. The 2.7g example is the same as Figure 3(a).

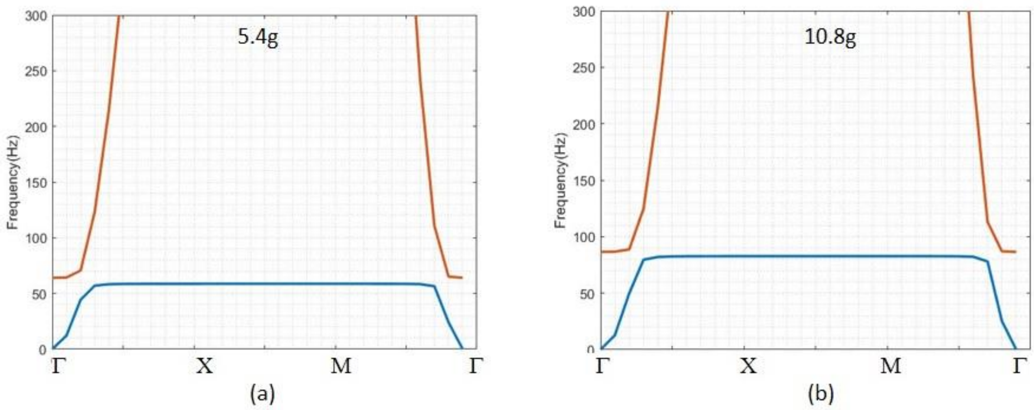

Figure 4. Bandgap structure of membrane-type metamaterial attached with: (a) 5.4g and (b) 10.8g mass. The bandgap range: (a) 82.7 – 86.6 Hz and (b) 58.7 – 64.1 Hz.

According to the figure, the bandgap location will grow significantly when the attached mass is decreased. That is similar to what we found in former working paper [13].

3.2 Finite structure and numerical simulation

Finite structure model is constructed to verify the accuracy the proposed theoretical method. As shown in Figure 5, a thin steel plate is attached with 5×5 unit cells of membrane-type metamaterial. The dimension of the thin plate is $500 \times 300 \times 2$mm, and the outer dimension of one unit cell is 60×60mm.

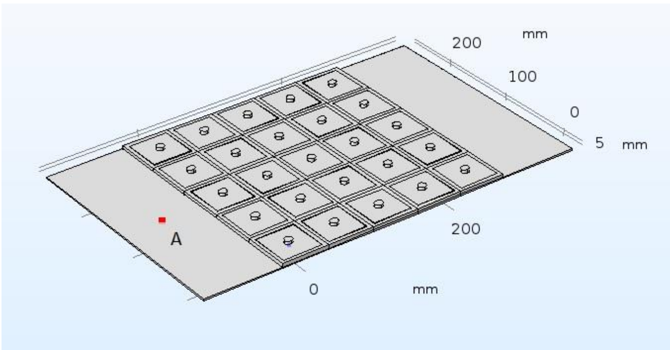

Figure 5. Configuration of finite structure

In order to examine the bandgap performance of the structure, the frequency domain analysis is conducted on the model. In the theoretical model, the structure is assumed as infinite. However, in actual application it is impossible so certain number of periodicity is required to let the finite structure generate bandgap behaviour. In this work we adopted 5 by 5 unit arrays which is enough for the bandgap forming. The left edge of the plate is fixed and transverse excitation signal input from the right edge. Other boundaries are set as free. Biaxial prestress condition is applied to membrane. The response signal is picked up from point A. The setting of the model is the same as mentioned in Table 1.

The frequency scanning range is set from 50 to 300 Hz and stress define as 2 MPa, 3 MPa, 6 MPa and 10 MPa respectively. The results are given in Figure 6.

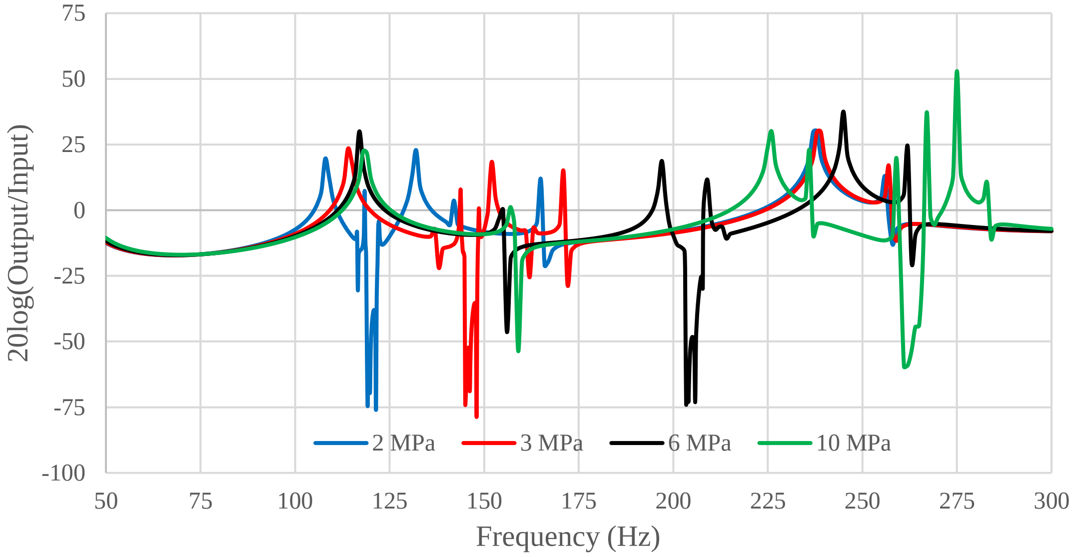

Figure 6. Frequency response of plate attached with membrane-type metamaterial. The response curves of the membrane resonator applied with different stress is presented by: Blue line (2MPa); Red line (3MPa); Black line (6MPa) and Green line (10MPa).

As shown in the figure, the bandgap ranges are: 118.4 – 121.8 Hz (2 MPa), 144.8 – 148.6 Hz (3 MPa), 203.4 – 208 (6MPa) and (10 MPa). It is obvious that the vibration transmission in the bandgap range is efficiently attenuated. Within membrane, the stiffness is mainly provided by stress. So the resonance frequency will increase along with stress. Also, the bandgap width is extended as well.

In addition, the effect of mass is also investigated. In the finite structure, the stress is kept at 2 MPa and define the mass as 2.7g and 10.8g respectively. The curves are given in Figure 7. The bandgap ranges are: 118.4 – 121.8 Hz (2.7g) and 59 – 66 Hz (10.8g). The results are similar to the theoretical results given in former section. The bandgap width increased with mass magnitude.

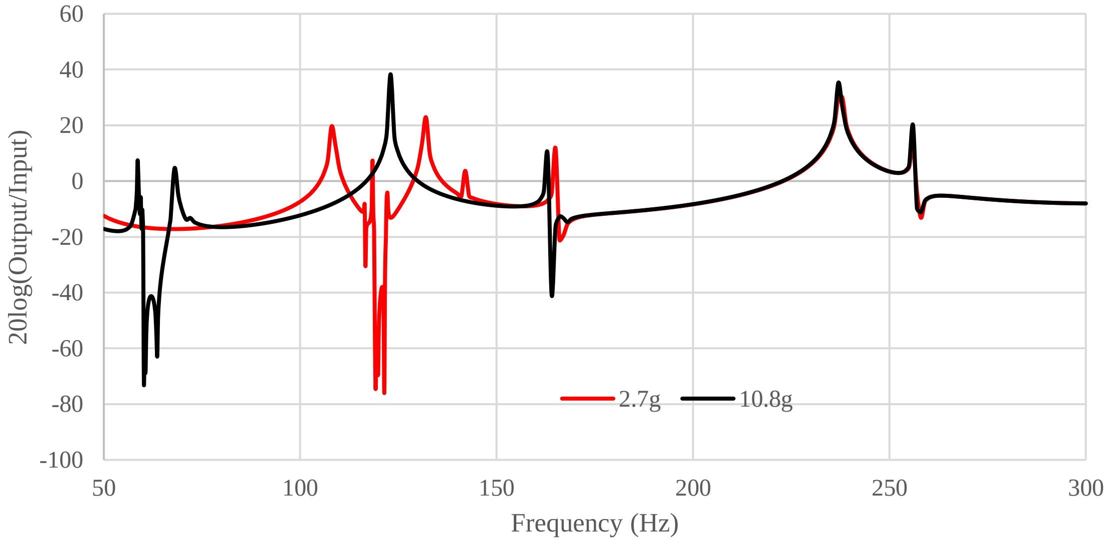

Figure 7. Frequency response of plate when the mass magnitude is 2.7g (Red line) and 10.8g (black line).

4. CONCLUSIONS

In this work we proposed a theoretical method to predict the bandgap properties of membrane-type metamaterial periodically attached to thin plate. By using the theoretical method, the effect of tuning stress level and attached mass magnitude on the membrane-type metamaterial is investigated. It is found that the increase of stress level will lead to rising of bandgap location and bandgap width. The increment of mass magnitude will have the same effect to bandgap width and counter effect to bandgap location. The theoretical model results are verified by numerical simulation and are basically consistent. Within the model, the parameters of membrane-type metamaterial can be adjusted directly and the model is able to present the corresponding bandgap change rapidly. Time consumption is significantly reduced compared with FEM. It is considered an effective tool for the design of membrane-type metamaterial.

5. ACKNOWLEDGEMENTS

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